Spatial Ability and Geometry Achievement: A Pilot Study

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Project Goals

To use performance on spatial tests to predict geometry success.

To develop an educational intervention to improve geometry achievement by training students' spatial skills.

Background

American Students Struggle with Geometry

1995 TIMSS: American 12th graders had the most difficulty with the geometry content area (Mullis, 1996) and scored the lowest of all countries in geometry (Wilson & Blank, 1999).

2003 TIMSS: American students' performance in geometry lagged behind performance in the other content areas; the average geometry subscore was nearly 30 points lower than the average composite score (Mullis, Martin, Gonzalez, & Chrostowski, 2004).

There was no detectable improvement in U. S. 8th-graders' geometry scores between 1999 and 2003; in contrast, there was a significant gain in U. S. 8th-graders' algebra scores during the same time period (Gonzales et al., 2004).

Geometry Supports Achievement in Higher-Level Mathematics

"Geometric thinking is an absolute necessity in every branch of mathematics, and, throughout history, the geometric point of view has provided exactly the right insight for many investigations…. For the past 150 years, the language of points, lines, angles, planes, surfaces, areas, and volumes has been applied to seemingly non-geometric phenomena, providing insight and coherence in many disparate branches of mathematics." (Cuoco, Goldberg, and Mark, 1996, p. 389)

Spatial Ability Supports Geometry Achievement

Many studies have found correlations between spatial ability and mathematics/geometry achievement (e.g., Battista, 1990; Battista, Wheatley, and Talsma, 1982; Connor and Serbin, 1985; Fennema & Sherman, 1977).

Several studies have found improvements in mathematics achievement following spatial training (e.g., Baldwin, 1984; Moses, 1977). The effects found in these studies were small, probably because of a poor match between training and test items.

Spatial Processing Presents a Significant Source of Difficulty in Geometry Problems

"…perceptual analysis and synthesis of mathematical information presented implicitly in a diagram often make greater demands on a pupil than any other aspect of the problem." (Lean and Clements, 1981, p. 277)

Geometrical Functional Fixedness (GFF) may explain why students often fail to extract important information from a diagram and thus struggle during problem-solving. For example, a student might be unable to see that a particular line segment is the height of a given triangle if it also happens to be the side of a different figure (Hoz, 1981).

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Pilot Study

Can we find evidence of spatial involvement in a geometry area task?

Method

Participants

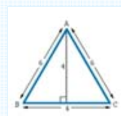
24 participants from the Carnegie Mellon undergraduate psychology participant pool. Most participants reported having taken Euclidean geometry in high school and introductory calculus in college.

Design

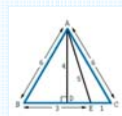
4 × 6 × 2 completely within-subjects factorial design.

Factors

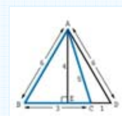
Problem Type



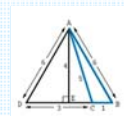
Simple



Complex3

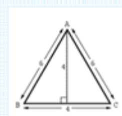


Complex2

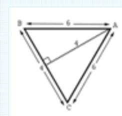


Complex1

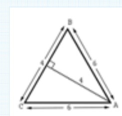
Degrees of Rotation



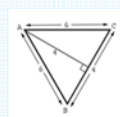
0°



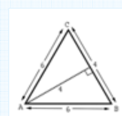
60°



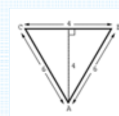
120°



300°

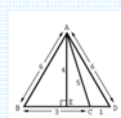


240°

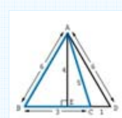


180°

Highlight



No Highlight



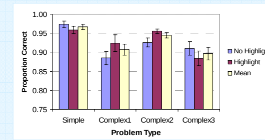
Yes Highlight

Procedure

After a brief warm-up and training procedure, participants computed the area of triangle ABC on the presented figures and entered their responses via computer keyboard. Problems timed out after 15 seconds. Accuracy and latency were recorded. Participants received feedback on their performance.

Results

Accuracy



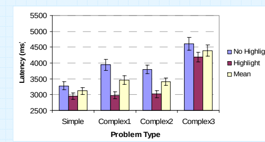
Main effect of Problem Type on proportion correct, $p < .001$.

Problem Type × Highlight interaction is significant at $p = .037$.

Highlight had a significant effect on Complex2 accuracy only.

Latency

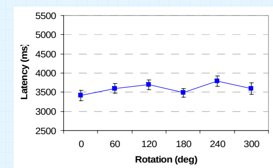
Latency analyses are performed on correct trials only.



Problem Type × Highlight interaction is significant at $p < .001$.

Participants are faster on Highlight problems, but the size of this effect depends on Problem Type.

Participants are fastest on Simple and slowest on Complex3 problems.

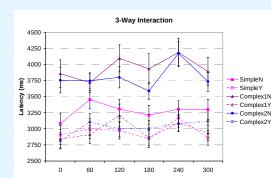


Main effect of Degrees of Rotation on latency, $p < .001$.

Participants are fastest at 0° and 180°, 60°/120°/300°, and slowest at 240° problems.

This suggests that mental rotation may be required to solve these problems.

Evidence for existence of an "embeddedness effect."



If we exclude Complex3 problems, which require an extra addition step, from the analysis and examine the 3-way interaction, we find an "embeddedness effect."

Participants are faster on problems that don't require them to extract an embedded figure—Simple and Highlighted problems.

Ongoing and Future Work

We will soon be piloting on our target population: high school geometry students. In this second pilot, we will be adding a battery of spatial tests and using participants' performance on these tests to predict effects on the area task.

We are also in the process of designing training materials for mental rotation and embedded figures tasks and will be piloting these materials later.

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